DAY — **09** SEAT NUMBER | | | | | | |

| 2025 | II | 22 | 1100 | **J-312** | (E) |
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# MATHEMATICS & STATISTICS (40)
# (ARTS & SCIENCE)

**Time : 3 Hrs.** **(8 Pages)** **Max. Marks : 80**

***General instructions :***

*The question paper is divided into **FOUR** sections.*

(1) ***Section A :*** *Q. 1 contains **Eight** multiple choice type questions carrying **Two** marks each.*

*Q. 2 contains **Four** very short answer type questions carrying **One** mark each.*

(2) ***Section B :*** *This section contains **Twelve** short answer type questions carrying **Two** marks each. (Attempt any **Eight**)*

(3) ***Section C :*** *This section contains **Twelve** short answer type questions carrying **Three** marks each. (Attempt any **Eight**)*

(4) ***Section D :*** *This section contains **Eight** long answer type questions carrying **Four** marks each. (Attempt any **Five**)*

(5) *Use of log table is allowed. Use of calculator is not allowed.*

(6) *Figures to the right indicate full marks.*

(7) *Use of graph paper is <u>not</u> necessary. Only rough sketch of graph is expected.*

(8) *For each multiple choice type of questions, only the first attempt will be considered for evaluation.*

(9) *Start answer to each section on a new page.*

# SECTION – A

**Q. 1. Select and write the correct answer of the following multiple choice type of questions :** **[16]**

(i) If $A = \{1, 2, 3, 4, 5\}$ then which of the following is not true ?

(a) $\exists x \in A$ such that $x + 3 = 8$

(b) $\exists x \in A$ such that $x + 2 < 9$

(c) $\forall x \in A,\ x + 6 \geq 9$

(d) $\exists x \in A$ such that $x + 6 < 10$ (2)

(ii) In $\Delta ABC$, $(a + b) \cdot \cos C + (b + c) \cos A + (c + a) \cdot \cos B$ is equal to _____.

(a) $a - b + c$

(b) $a + b - c$

(c) $a + b + c$

(d) $a - b - c$ (2)

(iii) If $|\bar{a}| = 5$, $|\bar{b}| = 13$ and $|\bar{a} \times \bar{b}| = 25$ then $|\bar{a} \cdot \bar{b}|$ is equal to _____.

(a) 30 (b) 60

(c) 40 (d) 45 (2)

(iv) The vector equation of the line passing through the point having position vector $4\hat{i}-\hat{j}+2\hat{k}$ and parallel to vector $-2\hat{i}-\hat{j}+\hat{k}$ is given by _____.

(a) $(4\hat{i}-\hat{j}-2\hat{k})+\lambda(-2\hat{i}-\hat{j}+\hat{k})$

(b) $(4\hat{i}-\hat{j}+2\hat{k})+\lambda(2\hat{i}-\hat{j}+\hat{k})$

(c) $(4\hat{i}-\hat{j}+2\hat{k})+\lambda(-2\hat{i}-\hat{j}-\hat{k})$

(d) $(4\hat{i}-\hat{j}+2\hat{k})+\lambda(-2\hat{i}-\hat{j}+\hat{k})$ (2)

(v) Let $f(1)=3$, $f'(1)=-\frac{1}{3}$, $g(1)=-4$ and $g'(1)=-\frac{8}{3}$. The derivative of $\sqrt{[f(x)]^2+[g(x)]^2}$ w.r.t. $x$ at $x=1$ is _____.

(a) $-\frac{29}{25}$ (b) $\frac{7}{3}$

(c) $\frac{31}{15}$ (d) $\frac{29}{15}$ (2)

(vi) If the mean and variance of a binomial distribution are 18 and 12 respectively, then $n$ is equal to _____.

(a) 36 (b) 54

(c) 18 (d) 27 (2)

(vii) The value of $\int x^x(1+\log x)\,dx$ is equal to _____.

(a) $\frac{1}{2}(1+\log x)^2+c$ (b) $x^{2x}+c$

(c) $x^x\cdot\log x+c$ (d) $x^x+c$ (2)

(viii) The area bounded by the line $y=x$, X-axis and the lines $x=-1$ and $x=4$ is equal to _____. (in square units)

(a) $\frac{2}{17}$ (b) 8

(c) $\frac{17}{2}$ (d) $\frac{1}{2}$ (2)

**Q. 2.** **Answer the following questions :** **[4]**

(i) Write the negation of the statement : '$\exists n \in N$ such that $n+8>11$' (1)

(ii) Write unit vector in the opposite direction to $\bar{u}=8\hat{i}+3\hat{j}-\hat{k}$. (1)

(iii) Write the order of the differential equation

$$\sqrt{1+\left(\frac{dy}{dx}\right)^2}=\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}}$$ (1)

(iv) Write the condition for the function $f(x)$, to be strictly increasing, for all $x \in R$. (1)

# SECTION – B

**Attempt any EIGHT of the following questions :** **[16]**

**Q. 3.** Using truth table, prove that the statement patterns $p \leftrightarrow q$ and $(p \wedge q) \vee (\sim p \wedge \sim q)$ are logically equivalent. (2)

**Q. 4.** Find the adjoint of the matrix $\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$. (2)

**Q. 5.** Find the general solution of $\tan^2\theta = 1$. (2)

**Q. 6.** Find the co-ordinates of the points of intersection of the lines represented by $x^2-y^2-2x+1=0$. (2)

**Q. 7.** A line makes angles of measure 45° and 60° with the positive directions of the $Y$ and $Z$ axes respectively. Find the angle made by the line with the positive direction of the X-axis. (2)

**Q. 8.** Find the vector equation of the plane passing through the point having position vector $2\hat{i}+3\hat{j}+4\hat{k}$ and perpendicular to the vector $2\hat{i}+\hat{j}-2\hat{k}$. (2)

**Q. 9.** Divide the number 20 into two parts such that sum of their squares is minimum. (2)

**Q. 10.** Evaluate : $\int x^9 . \sec^2(x^{10})dx$ (2)

**Q. 11.** Evaluate : $\int \frac{1}{25-9x^2}dx$ (2)

**Q. 12.** Evaluate : $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1-\sin x}dx$ (2)

**Q. 13.** Find the area of the region bounded by the parabola $y^2 = 16x$ and its latus rectum. (2)

**Q. 14.** Suppose that $X$ is waiting time in minutes for a bus and its p.d.f. is given by :

$f(x) = \frac{1}{5}$, for $0 \leq x \leq 5$

$= 0$, otherwise.

Find the probability that :

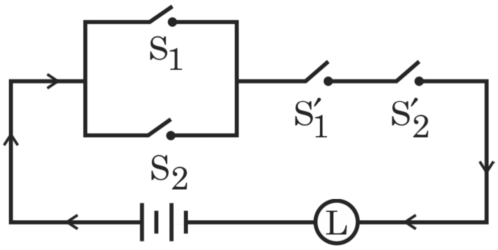
(i) waiting time is between 1 to 3 minutes.

(ii) waiting time is more than 4 minutes. (2)

# SECTION – C

**Attempt any EIGHT of the following questions :** **[24]**

**Q. 15.** Express the following switching circuit in the symbolic form of logic. Construct the switching table and interpret it :




(3)

**Q. 16.** Prove that : $2\tan^{-1}\left(\frac{1}{3}\right)+\cos^{-1}\left(\frac{3}{5}\right)=\frac{\pi}{2}$. (3)

**Q. 17.** In $\Delta ABC$ if $a$ = 13, $b$ = 14, $c$ = 15 then find the values of (i) $\sec A$ (ii) $\operatorname{cosec}\frac{A}{2}$. (3)

**Q. 18.** A line passes through the points (6, –7, –1) and (2, –3, 1). Find the direction ratios and the direction cosines of the line. Show that the line does not pass through the origin. (3)

**Q. 19.** Find the cartesian and vector equations of the line passing through $A$ (1, 2, 3) and having direction ratios 2, 3, 7. (3)

**Q. 20.** Find the vector equation of the plane passing through points $A$ (1, 1, 2), $B$ (0, 2, 3) and $C$ (4, 5, 6). (3)

**Q. 21.** Find the $n^{th}$ order derivative of $\log x$. (3)

**Q. 22.** The displacement of a particle at time $t$ is given by $s=2t^3-5t^2+4t-3$. Find the velocity and displacement at the time when the acceleration is 14 ft/sec$^2$. (3)

**Q. 23.** Find the equations of tangent and normal to the curve $y = 2x^3 - x^2 + 2$ at point $\left(\frac{1}{2}, 2\right)$. (3)

**Q. 24.** Three coins are tossed simultaneously, $X$ is the number of heads. Find the expected value and variance of $X$. (3)

**Q. 25.** Solve the differential equation : $x\frac{dy}{dx} = x \cdot \tan\left(\frac{y}{x}\right) + y$. (3)

**Q. 26.** Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Find the probability that :

(i) all the five cards are spades.

(ii) none is spade. (3)

# SECTION – D

**Attempt any FIVE of the following questions :** **[20]**

**Q. 27.** Find the inverse of $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ by elementary row transformations. (4)

**Q. 28.** Prove that homogeneous equation of degree two in $x$ and $y$, $ax^2 + 2hxy + by^2 = 0$ represents a pair of lines passing through the origin if $h^2 - ab \geq 0$. Hence show that equation $x^2 + y^2 = 0$ does not represent a pair of lines. (4)

**Q. 29.** Let $\bar{a}$ and $\bar{b}$ be non-collinear vectors. If vector $\bar{r}$ is coplanar with $\bar{a}$ and $\bar{b}$ then show that there exist unique scalars $t_1$ and $t_2$ such that $\bar{r} = t_1\bar{a} + t_2\bar{b}$. For $\bar{r} = 2\hat{i} + 7\hat{j} + 9\hat{k}$, $\bar{a} = \hat{i} + 2\hat{j}$, $\bar{b} = \hat{j} + 3\hat{k}$, find $t_1$, $t_2$. (4)

**Q. 30.** Solve the linear programming problem graphically.

Maximize : $z = 3x + 5y$

Subject to : $x + 4y \le 24,$

$3x + y \le 21,$

$x + y \le 9,$

$x \ge 0, y \ge 0$

Also find the maximum value of $z$. (4)

**Q. 31.** If $x = f(t)$ and $y = g(t)$ are differentiable functions of $t$ so that $y$ is a function of $x$ and if $\frac{dx}{dt} \ne 0$

then prove that $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$.

Hence find the derivative of $7^x$ w.r.t. $x^7$. (4)

**Q. 32.** Evaluate : $\int e^{\sin^{-1} x}\left(\frac{x + \sqrt{1 - x^2}}{\sqrt{1 - x^2}}\right)dx$ (4)

**Q. 33.** Prove that : $\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$

Hence evaluate : $\int_0^3 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{3 - x}}dx$ (4)

**Q. 34.** If a body cools from 80°C to 50°C at room temperature of 25°C in 30 minutes, find the temperature of the body after 1 hour. (4)

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